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BPS states with extra supersymmetry

Jerome P. Gauntlett and Chris M. Hull

Department of Physics, Queen Mary and Westfield College
University of London, Mile End Road, London E1 4NS, UK
E-mail: j.p.gauntlett@qmw.ac.uk, c.m.hull@qmw.ac.uk

Abstract: A state saturating a BPS bound derived from a supersymmetry algebra preserves some fraction of the supersymmetry. This fraction of supersymmetry depends on the charges carried by the system, and we show that in general there are configurations of charges for which a BPS state would preserve more than half the original supersymmetry. We investigate configurations that could preserve 3/4 supersymmetry in string theory, M-theory and supersymmetric field theories and discuss whether states saturating these bounds actually occur in these theories.

Keywords: M-Theory, p-branes.

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Contents

1. Introduction

It is well known that the supersymmetry algebra admits central charges that give BPS bounds on the energy. These charges can be carried by solitons and when the bound is saturated the states preserve some fraction of the supersymmetry. In addition there are tensorial 'central' charges carried by various p-branes in string/M-theory, for example, that lead to BPS bounds on the energy densities of the branes, and the BPS p-branes preserve 1/2 of the supersymmetry [1]. The p-branes can intersect with or end on other branes while still preserving some supersymmetry, and intersecting brane configurations have been found that preserve fractions $n/32$ of the supersymmetry for $n = 0, 1, 2, 3, 4, 5, 6, 8, 16$ so that in each of these cases no more than half the supersymmetry is preserved; see, for example, [2, 3, 4, 5, 6].

By examining the supersymmetry algebra it is simple to see that there must exist charges that would correspond to preservation of any fraction n/M of supersymmetry (where M is the number of supersymmetries of the system, so that $M = 32$ for M-theory). The general anticommutator of N supercharges $Q_{\alpha I}$ (with α a spinor index and $I = 1, \ldots, N$) can be written as

$$\{Q_A, Q_B\} = M_{AB}\,, \tag{1.1}$$

where $A = 1, \ldots, M$ is a composite index $A = \{\alpha I\}$ and M_{AB} is a symmetric matrix of bosonic charges, which in most physical systems will take the form

$$M_{AB} = H\delta_{AB} - Z_{AB}\,, \tag{1.2}$$

with H the hamiltonian and Z_{AB} a traceless symmetric matrix of 'central' charges which can be decomposed into a set of p-form charges $Z^{IJ}_{\mu_1 \ldots \mu_p}$ contracted with gamma matrices. Let the eigenvalues of Z_{AB} be $\lambda_1, \ldots, \lambda_M$ with $\sum \lambda_A = 0$. Then the supersymmetry algebra implies that M_{AB} must be positive definite so that the energy E is bounded below by the largest eigenvalue, $E \geq \lambda$ where $\lambda = max\{\lambda_1, \dots, \lambda_M\}$, as is easily seen in a basis in which Z_{AB} is diagonal. If the largest eigenvalue is n-fold degenerate, $\lambda_1 = \lambda_2 = \cdots = \lambda_n \equiv \lambda$ say, and if there is a state that saturates the bound with $E = \lambda$, then for this state M_{AB} will have n zero eigenvalues and by definition the state will preserve n of the supersymmetries, namely $Q_1, Q_2, \dots, Q_n$, and should fit into a supermultiplet generated by the action of the remaining $M - n$ supercharges. Thus a system will have a state preserving a given fraction n/M of supersymmetry provided (i) there is a configuration of charges such that the maximal eigenvalue of Z_{AB} is n-fold degenerate and (ii) there is a state that saturates the BPS bound for these charges.

In many physical systems, Z_{AB} is an arbitrary symmetric traceless matrix, since a configuration of charges can be found that gives any desired Z_{AB}. For example, in M-theory the matrix M_{AB} has $32 \times 33/2 = 528$ independent entries and all 528 arise from the 11-momentum, a 2-form charge and a 5-form charge, as 11+55+462=528 [7]. Moreover, each of the 527 charges Z_{AB} is believed to actually arise in M-theory, and there is a 1/2-supersymmetric BPS state for each of the 527 charges [8]. Most have been constructed explicitly, while evidence for the occurrence of the M9-brane is given in [9, 10]. Then in M-theory there is a configuration of charges corresponding to each fraction $n/32$ of supersymmetry for $0 \leq n \leq 32$, and most can be realised without recourse to M9-branes.

If M-theory is dimensionally reduced to one dimension by compactifying all the spatial dimensions, the resulting theory is a quantum mechanical theory with 32 supersymmetries and algebra (1.1), (1.2), where $A = 1, \dots, 32$ is now an internal index transforming under an Sp(32) internal symmetry, and Z_{AB} represents 527 scalar central charges, transforming irreducibly under the Sp(32) automorphism group of the superalgebra, which is a contraction of OSp(32|1). All central charges are then clearly on the same footing, and there seems no reason why an arbitrary central charge matrix Z_{AB}, and hence an arbitrary fraction of supersymmetry $n/32$, should not be realisable.

If there is a set of charges in a supersymmetric theory for which the maximal eigenvalue of Z_{AB} is n-fold degenerate, and if there is a state which saturates the BPS bound, it would preserve n/M of the supersymmetries. In most cases that have been studied and for which the state of lowest energy has been found, it turns out to be a supersymmetric one saturating the bound. The fact that most allowed supersymmetric states actually occur suggests that it would be of interest to investigate further the configurations that could preserve exotic fractions of supersymmetry.

Our purpose here will be to give some simple examples in which there is a BPS bound for which a state saturating it would preserve 3/4 supersymmetry, and to give some preliminary discussion as to whether such states actually occur. The possibility of 3/4 supersymmetry has also been recently discussed in [11, 12].

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We will first consider the supersymmetry algebra in four dimensions. It is straightforward to provide charges that lead to preservation of 3/4 of the supersymmetry. This algebraic structure can be embedded in higher dimensions and we will focus on D=11. We will show that the charges preserving 3/4 of the supersymmetry can be realised by considering a very simple configuration of a membrane intersecting two fivebranes according to the array

$$\begin{array}{lcccccccccc} M5: & 1 & 2 & 3 & 4 & 5 & & & & & \\ M5: & 1 & & & & & 6 & 7 & 8 & 9 & \\ M2: & 1 & & & & & & & & & \sharp \end{array} \tag{1.3}$$

where the symbol $\sharp$ is read as '10', with the amount of supersymmetry preserved depending on the energy and the charges of the three branes. The case that has been discussed previously [3, 4] is that in which the product of all three brane charges is positive (in our conventions), leading to 1/4 supersymmetry being preserved, whereas we will find new possibilities when one of the branes has negative charge, and so is an anti-brane (or all three are anti-branes). We will analyse this case in some detail and determine under which conditions the fractions 1/4, 1/2 and 3/4 of the supersymmetry could be preserved. One interesting feature is that for three or more intersecting branes, switching all the branes to anti-branes can lead to inequivalent results, whereas for configurations with just two branes, equivalent results would be obtained by the switch. Many other configurations of branes with exotic supersymmetry in M-theory or string theory can be generated from this example by dualities.

2. Exotic supersymmetry in $D = 4$

The general N extended superalgebra in four dimensions is

$$\{Q^I, \bar{Q}^J\} = -\Big(P^\mu \delta^{IJ}\Gamma_\mu + V^{IJ}_\mu \Gamma^\mu + iY^{IJ}_\mu \Gamma^5\Gamma^\mu + X^{IJ}_{\mu\nu}\Gamma^{\mu\nu} + iZ^{IJ} + i\tilde{Z}^{IJ}\Gamma^5\Big), \tag{2.1}$$

where Q^I, $I = 1, \ldots, N$ is a Majorana spinor, the charges $Y^{IJ}_\mu, Z^{IJ}, \tilde{Z}^{IJ}$ are antisymmetric in the IJ indices while $V^{IJ}_\mu, X^{IJ}_{\mu\nu}$ are symmetric and V^{IJ}_μ is traceless. In supersymmetric theories, P_μ is the 4-momentum, Z and $\tilde{Z}$ are electric and magnetic 0-brane charges, $X_{\mu\nu}$ are domain wall charges [13], V_i, Y_i are string charges ($i = 1, 2, 3$) and V_0, Y_0 are charges for space-filling 3-branes [8]. Moreover, in some cases P_i could be a linear combination of the momentum and a string charge, while P^0 could be a linear combination of the energy and a 3-brane charge.

The number of charges on the right-hand-side of (2.1) is $10 \times N(N+1)/2 + 6 \times N(N-1)/2$, which agrees with the number of components, $2N(4N+1)$, of the left-hand-side. This suggests that by choosing the charges on the right hand side, it should be possible to find a system for which the BPS bound would lead to any fraction $n/4N$ supersymmetry being preserved, provided that there existed a state saturating the BPS bound. In particular, there are some very simple systems that could allow 3/4 supersymmetry.

For example, consider $N = 2$ supersymmetry with only the charges P_μ and $Y_\mu^{IJ} = Y_\mu \epsilon^{IJ}$ non-zero. A convenient choice of gamma matrices is

$$\Gamma^0 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \qquad \Gamma^i = \begin{pmatrix} 0 & i\sigma^i \\ -i\sigma^i & 0 \end{pmatrix}, \qquad \Gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.2}$$

Then configurations with $P^0 = E$, $P^3 = p$, $Y_0 = u$ and $Y_3 = v$ and all other charges zero have the superalgebra

$$\{Q, Q^\dagger\} = \mathrm{diag}(E - \lambda_1, E - \lambda_2, E - \lambda_3, E - \lambda_4)\,, \tag{2.3}$$

where $Q = (Q^1 + iQ^2)/\sqrt{2}$ and the eigenvalues λ_i are given by

$$\begin{aligned} \lambda_1 &= p + u + v\,, \\ \lambda_2 &= u - p - v\,, \\ \lambda_3 &= v - u - p\,, \\ \lambda_4 &= p - u - v\,. \end{aligned} \tag{2.4}$$

Note that there is a symmetry in the way the three charges occur. Positivity implies that the energy E satisfies $E \geq \lambda_i$ for each i. If only one of the charges is non-zero, u say, then $E \geq u$ and $E \geq -u$ so that we obtain the standard bound $E \geq |u|$. With two charges, u and v say, we obtain $E \geq |u + v|$ and $E \geq |u - v|$ and when one of these is saturated we have a configuration preserving 1/4 supersymmetry. With all three charges, there are four bounds corresponding to the four eigenvalues and in general when one is saturated there will be 1/4 supersymmetry preserved. However, for special values of the charges there can be degenerate eigenvalues. Consider for example the case in which all charges are equal, $u = v = p = -\lambda$ so that

$$\{Q, Q\} = \mathrm{diag}(H + 3\lambda, H - \lambda, H - \lambda, H - \lambda)\,. \tag{2.5}$$

If λ is positive, a state with $E = \lambda$ would preserve the 3/4 supersymmetry corresponding to supersymmetry parameters of the form $\epsilon = (0, \epsilon_2, \epsilon_3, \epsilon_4)$. For negative λ, a BPS state with $E = -3\lambda$ would preserve 1/4 supersymmetry.

In [14], it will be shown that a similar example occurs in the Wess-Zumino model with $N = 1$ supersymmetry. In that case, there is again a simple configuration, corresponding to intersecting domain walls with momentum along the intersection, for which a state saturating the bounds would have 1/4, 1/2 or 3/4 supersymmetry, depending on the values of the charges. It will also be shown in [14] that the Wess-Zumino model does not admit any classical configurations with 3/4 supersymmetry.

3. Exotic supersymmetry in string theory and M-theory

3.1 M-theory

The general form of the eleven dimensional supersymmetry algebra has

$$\{Q,Q\} = C\left(\Gamma^M P_M - \frac{1}{2!}\Gamma^{M_1M_2}Z_{M_1M_2} - \frac{1}{5!}\Gamma^{M_1\ldots M_5}Z_{M_1\ldots M_5}\right), \tag{3.1}$$

where C is the charge conjugation matrix, P_M is the energy-momentum 11-vector and $Z_{M_1M_2}$ and $Z_{M_1\ldots M_5}$ are 2-form and 5-form charges. The fraction of supersymmetry that is preserved by a configuration possessing a given set of charges is given by the number of zero eigenvalues of the matrix $\{Q,Q\}$ divided by 32. As argued in the introduction, both sides have equal numbers of components (528), and all 528 charges on the right hand side actually arise in M-theory, provided we include M9-branes carrying the charge Z_{0i} [8], so that there must be configurations of charges that could give rise to all fractions $n/32$ of preserved supersymmetry, provided BPS states arise in that charge sector.

For 3/4 supersymmetry, there is a very simple set of charges corresponding to two fivebranes and a membrane that allows 3/4 of the supersymmetry, obtained by embedding the example of the last section in 11 dimensions and using dualities. In addition we will show that there are some novel combinations of three charges leading to the preservation of 1/4 and 1/2 supersymmetry. It is known [3, 4] that it is possible to have two fivebranes and a membrane intersecting according to (1.3) and preserving 1/4 of the supersymmetry, provided the product of all three brane charges is positive. Changing the signs of one or three of the charges and tuning their values allows 3/4 supersymmetry instead, as we shall see.

We begin by assuming that the only non-zero charges in (3.1) are

$$\begin{aligned} q_5 &= Z_{12345}\,,\\ q_5' &= Z_{16789\sharp}\,,\\ q_2 &= Z_{1\sharp} \end{aligned} \tag{3.2}$$

and positive charges will correspond to branes and negative charges to anti-branes. We use real gamma matrices with $C=\Gamma^0$ and $\Gamma^{0123456789\sharp}=1$. It will be convenient to take a basis such that

$$\begin{aligned} \Gamma^{012345} &= \mathrm{diag}(1,1,-1,-1)\otimes \mathbb{1}_8\,,\\ \Gamma^{016789} &= \mathrm{diag}(1,-1,1,-1)\otimes \mathbb{1}_8\,,\\ \Gamma^{01\sharp} &= \mathrm{diag}(1,-1,-1,1)\otimes \mathbb{1}_8\,, \end{aligned} \tag{3.3}$$

where $\mathbb{1}_8$ is the 8×8 identity matrix. Setting $P^0=E$ we can then rewrite (3.1) as

$$\{Q,Q\} = \mathrm{diag}(E-\lambda_1, E-\lambda_2, E-\lambda_3, E-\lambda_4)\otimes \mathbb{1}_8\,, \tag{3.4}$$

where

$$
\begin{aligned}
\lambda_1 &= q_2 + q_5 + q_5' \,, \\
\lambda_2 &= -q_2 + q_5 - q_5' \,, \\
\lambda_3 &= -q_2 - q_5 + q_5' \,, \\
\lambda_4 &= q_2 - q_5 - q_5' \,.
\end{aligned} \tag{3.5}
$$

Since $\{Q, Q\}$ is a positive matrix we have the BPS bound $E \geq \lambda_i$ for $i = 1, 2, 3, 4$.

If there is only one non-zero charge, q_2 say, then the BPS bound is simply $E \geq |q_2|$ and when it is saturated 1/2 of the supersymmetry is preserved. For example, for BPS membranes (with q_2 positive and $E = q_2$) we have

$$\{Q, Q\} = \mathrm{diag}(0, 2q_2, 2q_2, 0) \otimes \mathbb{1}_8 \,. \tag{3.6}$$

The preserved supersymmetry parameters satisfy $\Gamma^{01\sharp}\epsilon = \epsilon$.

With an additional non-zero charge q_5, the BPS bounds are the two conditions that $E \geq |q_2 + q_5|$ and $E \geq |q_2 - q_5|$. When either of the bounds is saturated, 1/4 of the supersymmetry is preserved. For example, for a membrane and a fivebrane,

$$\{Q, Q\} = \mathrm{diag}(0, 2q_2, 2(q_2 + q_5), 2q_5) \otimes \mathbb{1}_8 \,, \tag{3.7}$$

with 8 zero eigenvalues. The supersymmetry preserved is the intersection of that preserved by each of the membranes and fivebranes; in this case

$$\Gamma^{01\sharp}\epsilon = \Gamma^{012345}\epsilon = \epsilon \,. \tag{3.8}$$

Adding the fivebrane to the membrane further halved the membrane's supersymmetries to leave 1/4 supersymmetry. However, a second fivebrane can now be added in the 16789 directions without breaking any more supersymmetry, as the corresponding projection $\Gamma^{016789}\epsilon = \epsilon$ on the supersymmetry parameter is already implied by the conditions (3.8). We can indeed add a third positive charge q_5' and preserve all 8 supersymmetries if the energy saturates the BPS bound, $E = q_2 + q_5 + q_5'$. We will refer to this as the usual BPS intersection of the $(2, 5, 5)$ system as it has been extensively studied in the literature. An identical analysis goes through for $(2, \bar{5}, \bar{5}')$ if we take $E = q_2 - q_5 - q_5'$, for $(\bar{2}, 5, \bar{5}')$ if we take $E = -q_2 + q_5 - q_5'$ and for $(\bar{2}, \bar{5}, 5')$ if we take $E = -q_2 - q_5 + q_5'$; in each case, we can start with any two of the branes intersecting and preserving 1/4 supersymmetry, and then add the third for free without any further breaking.

Returning to the $(2, 5)$ system preserving 8 supersymmetries, adding an anti-fivebrane with $\Gamma^{016789}\epsilon = -\epsilon$ instead of a fivebrane to give the $(2, 5, \bar{5}')$ configuration would appear to break the original 8 supersymmetries of the membrane-fivebrane system, but, as we shall show, the BPS bound leads to 8 supersymmetries if the energy saturates the bound. (These will be a different 8-dimensional subset of the 32 for some values of the charges and will be the same 8 for other values.) The situation is the same for the $(\bar{2},\bar{5},\bar{5}')$, $(\bar{2},5,5')$ and $(2,\bar{5},5')$ systems; in each case any two of the three branes preserve 8 supersymmetries, while the third brane appears to break these 8 supersymmetries, but nonetheless 8 supersymmetries would be preserved if the bound is saturated. Moreover, if such a 1/4 supersymmetric BPS state exists, tuning the charges to particular values enhances the number of supersymmetries to 16 or to the exotic value of 24.

For general charges it is useful to contrast the analysis for configurations related by switching branes with anti-branes and we will focus on the $(2,5,5')$ and $(\bar{2},\bar{5},\bar{5}')$ systems. With this in mind we return to (3.4) and (3.5) and first consider the $(2,5,5')$ case in which all the charges are positive. Clearly $\lambda_1 = q_2 + q_5 + q_5'$ is the largest eigenvalue and hence the BPS bound is $E \geq q_2 + q_5 + q_5'$ and when it is saturated we preserve 1/4 of the supersymmetry; this is the usual case considered above. To analyse the $(\bar{2},\bar{5},\bar{5}')$ case in which all charges are negative, it is useful to rewrite (3.5) as

$$\begin{aligned}
\lambda_1 &= q_2 + q_5 + q_5' \,, \\
\lambda_2 &= -(q_2 + q_5 + q_5') + 2q_5 \,, \\
\lambda_3 &= -(q_2 + q_5 + q_5') + 2q_5' \,, \\
\lambda_4 &= -(q_2 + q_5 + q_5') + 2q_2 \,.
\end{aligned} \tag{3.9}$$

One of $\lambda_2, \lambda_3, \lambda_4$ is now the biggest eigenvalue and is positive, since λ_1 is negative and the sum of the eigenvalues is zero. For example, when

$$0 \geq q_5 \geq q_5' \,, \qquad 0 \geq q_5 \geq q_2 \,, \tag{3.10}$$

it is λ_2 that is the biggest and the BPS bound is $E \geq -(q_2 + q_5 + q_5') + 2q_5$. If this bound is saturated

$$\{Q, Q\} = \mathrm{diag}\left(-2(q_2 + q_5'), 0, 2(q_5 - q_5'), 2(q_5 - q_2)\right) \otimes \mathbb{1}_8 \tag{3.11}$$

and 1/4 of the supersymmetry would be preserved. To obtain exotic preservation of 1/2 or 3/4 supersymmetry we only need to tune the charges: 1/2 supersymmetry is preserved when either $q_5 = q_5'$ or $q_5 = q_2$ and 3/4 supersymmetry is preserved when $q_5 = q_5' = q_2$.

Thus, for the $(\bar{2},\bar{5},\bar{5}')$ system with charges satisfying (3.10), the lowest energy allowed by supersymmetry is $E = -(q_2 + q_5 + q_5') + 2q_5$. If the ground state of this system indeed has this energy, then it would preserve 1/4 supersymmetry for generic values of the charges, but when two of the charges are equal, 1/2 supersymmetry would be preserved and if all three charges are equal, 3/4 supersymmetry would be preserved. Similar results follow for the cases in which it is λ_3 or λ_4 that is the biggest.

To obtain further insight, we continue with the case with charges satisfying (3.10) so that λ_2 is the largest eigenvalue. The specific 8 supersymmetries that would be preserved are the same as those preserved by the two intersecting branes $(\bar{2}, \bar{5}')$. It is interesting that it is these two branes that are contributing to the energy positively while the other fivebrane is contributing negatively. Recall that if we add a fivebrane, with q_5 positive, to $(\bar{2}, \bar{5}')$ to obtain the $(\bar{2}, 5, \bar{5}')$ configuration, the usual case of preservation of 1/4 supersymmetry is obtained if we have $E = -(q_2 + q_5') + q_5$. The new point here is that we can add instead an anti-fivebrane, with q_5 negative, to get the $(\bar{2}, \bar{5}, \bar{5}')$ system and still preserve the same supersymmetry if again $E = -(q_2 + q_5') + q_5$, as long as $q_5 \geq q_5'$ and $q_5 \geq q_2$. In either case, the naive energy would just be the sum of the energies of the three branes, i.e. $E_n = |q_2| + |q_5| + |q_5'|$. This is the correct result for the usual 1/4 supersymmetric $(\bar{2}, 5, \bar{5}')$ case, but for the exotic $(\bar{2}, \bar{5}, \bar{5}')$ case the energy of a state saturating the bound would be $E = E_n - V$ where $V = 2|q_5|$, suggesting that V might be interpreted as some kind of binding energy or as some tachyonic contribution. Note that the known supergravity solutions (with positive mass parameters) corresponding to $(\bar{2}, \bar{5}, \bar{5})$ have $E = |q_2| + |q_5| + |q_5'|$ which is greater than the BPS energy for these charges and so these states break all of the supersymmetry.

Finally, note that if the conditions (3.10) are not both satisfied, then either λ_3 or λ_4 will be the largest eigenvalue and adding the anti-fivebrane to $(\bar{2}, \bar{5}')$ will break the original 8 supersymmetries and lead to a *different* 8 supersymmetries being preserved.

3.2 Tachyon condensation

It is perhaps worth comparing the above with the case of coincident brane/anti-brane pairs. It has been argued that m D-branes and m anti-D-branes will completely annihilate to leave the vacuum with energy $E = mT + mT - V = 0$ where T is the energy of a single brane and the contribution $V = 2mT$ arises from the negative potential energy released by tachyon condensation [15, 16]. Duality then implies that this should also apply to any m brane/anti-brane pairs in M-theory or string theory, which should again completely annihilate. The tachyon condensation reduces the energy to the minimum allowed by the BPS bound, which in this case is zero as the brane/anti-brane pair carries no net charge. Adding a further n branes to obtain $n + m$ branes and m anti-branes, the m anti-branes should completely annihilate m of the branes to leave n branes with energy $E = nT$, which can be written as $E = E_n - V$ where $E_n = (2m + n)T$ is the naive energy given by the sums of the energies of the individual branes and anti-branes and $V = 2mT$.

Then for two coincident p-branes of charges q, $\tilde{q}$, the naive energy of the system would be the sum of the energies of the respective branes, $E_n = |q| + |\tilde{q}|$. This is the correct energy if $q, \tilde{q}$ have the same sign, so that they are either both branes or both anti-branes. However, if the charges have opposite sign so that one is a brane and the other an anti-brane (e.g. $q = (n + m)T$ and $\tilde{q} = -mT$ for the case above), the resulting configuration has $E = |q + \tilde{q}| = E_n - V$ with $V = 2 \min(|q|, |\tilde{q}|)$.

This is suggestively similar to the case considered above when λ_2 is the largest eigenvalue. The $(\bar{2},\bar{5}')$ system enters the energy formulae in exactly the same way as a 5-brane of charge $\tilde{q}_5 = -(q_2 + q_5')$ would. Adding a fivebrane with positive charge q_5 gives a system with $E = q_5 + \tilde{q}_5$, while adding an anti-fivebrane with negative charge $q_5 = -q$ gives a system with $E = \tilde{q}_5 - q$ and $V = 2q$. More generally, when the product of the brane charges is positive, the naive energy is the sum of the energies of the branes $E_n = |q_2| + |q_5| + |q_5'|$ and such configurations preserve 1/4 of the supersymmetry. When the product of the charges is negative, exotic preservation of supersymmetry is possible only if the naive energy is modified to $E = E_n - 2Min(|q_2|, |q_5|, |q_5'|)$. This suggests that tachyon condensation could play a role here also, reducing the energy below the sum of the brane energies. It seems plausible that this could indeed be the case and that it reduces the energy to the minimum allowed by supersymmetry.

3.3 String-theory

The M-theory example with an M2-brane and two M5-branes is related by duality to many other configurations of three branes in string theory or M-theory. For example, it is related to the type II configuration of a Dp-brane, a D$(8-p)$-brane and a fundamental string intersecting in a point, with the Dp-brane in the directions $1, 2, \ldots, p$, the D$(8-p)$-brane in the directions $p+1, p+2, \ldots, 8$ and the fundamental string in the 9th direction, or to the configuration of a D5-brane in the 12345 directions, a NS5-brane in the 12678 directions and a D3-brane in the 129 directions studied by Hanany and Witten [17]. In each case there is the usual 1/4 supersymmetric configuration in which one of the three branes is added 'for free', and an exotic configuration obtained from this by reversing the orientation of one of the branes, or of all three branes, in which a BPS state would preserve 1/4 supersymmetry for generic charges and 1/2 or 3/4 supersymmetry when two or three of the charges are of equal magnitude. There are no such configurations with only D-branes, so that the methods of [15, 16] cannot directly be used to test the possibility of tachyon condensation leading to exotic BPS states.

4. Conclusion

We have shown that the supersymmetry algebra allows configurations preserving exotic amounts of supersymmetry and we have identified simple configurations of charges in M-theory and in field theories such that any state with the lowest energy allowed by supersymmetry would preserve 3/4 supersymmetry, but we have not been able to establish whether or not such states actually occur. We have been unable to find any D=11 supergravity solutions with 24 Killing spinors, corresponding to 3/4 supersymmetry. The known supersymmetric supergravity solutions with two (anti)-fivebranes and an (anti)-membrane either preserve 1/4 or none of the su-

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persymmetry [3, 4]. We take the mass parameters associated with the harmonic or generalised harmonic functions of each of the individual branes to be positive. Then if the product of the three charges is positive (for example, the $(2, 5, 5')$ or $(2, \bar{5}, \bar{5}')$ configurations), then the solutions have 8 Killing spinors corresponding to 1/4 supersymmetry. When the product of the charges is negative, (for example, the $(\bar{2}, \bar{5}, \bar{5}')$ configuration), the known supergravity solutions actually break all of the supersymmetry. If we change the signs of some of the mass parameters we do not obtain solutions with more Killing spinors.[1] It is also possible to prove that no 3/4 supersymmetric classical solutions of the Wess-Zumino model exist, even though supersymmetry would have allowed them [14].

In each of our examples, there are no spatial dimensions that are transverse to all the branes and in such situations a number of subtleties can arise, but nonetheless our analysis does recover the known cases of supersymmetric intersections. The configurations we have identified are parameterised by three charges. For generic values of these charges, a BPS state would preserve 1/4 supersymmetry, but for special values when two or three of the charges are equal, a BPS state would preserve 1/2 or 3/4 supersymmetry, respectively.

We have conjectured that in string theory and M-theory, tachyon condensation could play a role in reducing the energy to the minimum allowed by the BPS bound, just as it does for the brane/anti-brane pair.

It would be very interesting to either establish that such exotic states do occur in certain theories, or, if they don't exist, to understand the reason for this, given that supersymmetry appears to allow them. We hope to return to these issues in the future.

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